The Mechanics of Tubular Fiber: Theoretical Analysis

DINESH K. GUPTA, *Department of Textile Engineering, Auburn University, Auburn, Alabama 36849*

Synopsis

A mathematical model of a tubular fiber is analyzed to predict its rigidity and some of its physical properties which are essential in textiles. The change in the properties of a tubular fiber with increase in the size of hollow core is considered and compared with the cylindrical fiber of the same outside diameter, the same denier, and the same rigidity as that of the tubular fiber. The equations are developed in dimensionless quantities to make them invariant to the units of fiber dimensions. The analysis revealed that having a hollow core of 4/10th the size of the outside diameter, in a solid fiber, could reduce the fiber weight by 16% without making any significant change in the rigidity of the fiber.

INTRODUCTION

About 70% of the total fibers used are man-made which can be engineered to satisfy a specific end use.[1] Most of the man-made fibers are produced from synthesis of petrochemicals such as polyester, nylon, acrylic, polypropylene, etc. About 2×10^8 lb of these fibers, called fiberfill, are used annually for stuffing in textile products, such as pillows, mattresses, sleeping bags, etc.[2] A large proportion of fibers is also used for packings and insulation. If the fibers used as fiberfill are tubular, then the same job of filling can be achieved by less fiber weight, which would not only reduce the weight of product and enhance its thermal insulation property, if desired, but would also save petroleum, which is an important source of energy.

At present, different deniers of synthetic fibers are produced for fiberfill. Six denier fibers are used as filling fibers in pillows, 15 denier fibers in furniture, and 40 denier for filtration purposes.[2] These fibers are 2–2.5 in. in staple length and made of polyester materials. In some cases the central core is made hollow which is approximately 15% of the total cross-sectional area of the fiber. The use of such fibers gives a 10–12% saving of the material. The fibers are crimped by stuffer-box technique to give them more bulk. Some fibers have their surface modified to give low stiffness properties to the product.

Use of tubular fiber in place of cylindrical fiber would affect the physical and mechanical properties of a product.[3–5] Tubular fiber has higher bulk, crimp, covering power, thermal insulation, and absorbing capacity in comparison to cylindrical fiber.[3] Short tubular fibers will have more crimp because of the difference in stresses on the inner and outer walls of the fiber tube during extrusion. The use of tubular fibers in sleeping bags showed better thermal insulation as compared to the use of cylindrical fibers.[4] This increase in insulation was due to the amount of dead-air trapped in the tubular fibers. Study on the

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use of tubular fibers in nonwovens showed that tubular fibers have good covering power because of more surface area per unit volume of the fiber.[5] Further study on the collapsibility of the fiber showed that it is dependent upon the ratio of the hollow to solid area, the elastic modulus of the material, and dimensions of the annular ring of tubular fibers. Using Timoshenko's theory of elastic stability, the critical pressure required to collapse a tubular fiber is defined as in Ref. 5:

$$\sigma_c = 2E[2t/(d_i + d_o)]^3 \tag{1}$$

where E is the elastic modulus, t is the wall thickness of fiber tube, d_i is the inner diameter of fiber tube, and d_o is the outside diameter of fiber tube.

The objective of this study was to analyze the effect of the size of hollow core on various properties of the fiber such as change in rigidity, weight, volume, etc. Three cases of a tubular fiber are analyzed and compared with one another for their various properties. The three cases of the tubular fibers are: fibers having constant outer diameter, constant denier (linear density), and constant rigidity. A definite relationship has been discovered to produce tubular fibers of different dimensions but having the same rigidity or stiffness property.

THEORY

Assumptions: The following assumptions are made to analyze the mathematical model of tubular fiber: (1) fibers are circular in cross section; (2) hollow core in tubular fiber is circular; (3) fibers are made of isotropic material; (4) fibers are subjected to small deformation; (5) fibers are subjected to either pure bending or pure twisting.

Nomenclature: The symbols used to analyze the mechanical and physical properties of a tubular fiber with respect to a cylindrical fiber are given in the Appendix.

Diameter of a Cylindrical Fiber: Consider a circular cylindrical fiber of diameter D(mm), and let ρ be the density of fiber material (g/cc); then the mass of 1 cm long fiber can be written as

$$w = (\rho \pi D^2/400)\ \mathrm{g} \tag{2}$$

If n_d is the linear density of the fiber [denier, i.e., (g) of 9000-m-long fiber], then eq. (2) can be written as

$$n_d = (\rho \pi D^2/400) \times 9 \times 10^6 \tag{3}$$

Simplifying and rearranging the terms in eq. (3) for the diameter D of the fiber gives

$$D = 0.0119\sqrt{n_d/\rho} \tag{4}$$

If the linear density of the fiber is given in tex, n_t, which is the weight (g) of 1000-m-long fiber, then eq. (4) can be modified to

$$D = 0.0357\sqrt{n_t/\rho} \tag{5}$$

Most of the textile fibers have material densities in the range of 0.9–2.7 g/cc.[6] The plot between the diameter of a circular cylindrical fiber and its denier value for different densities of polymeric material is shown in Figure 1. It shows a parabolic relation between the two parameters. As obvious from Figure 1, fibers

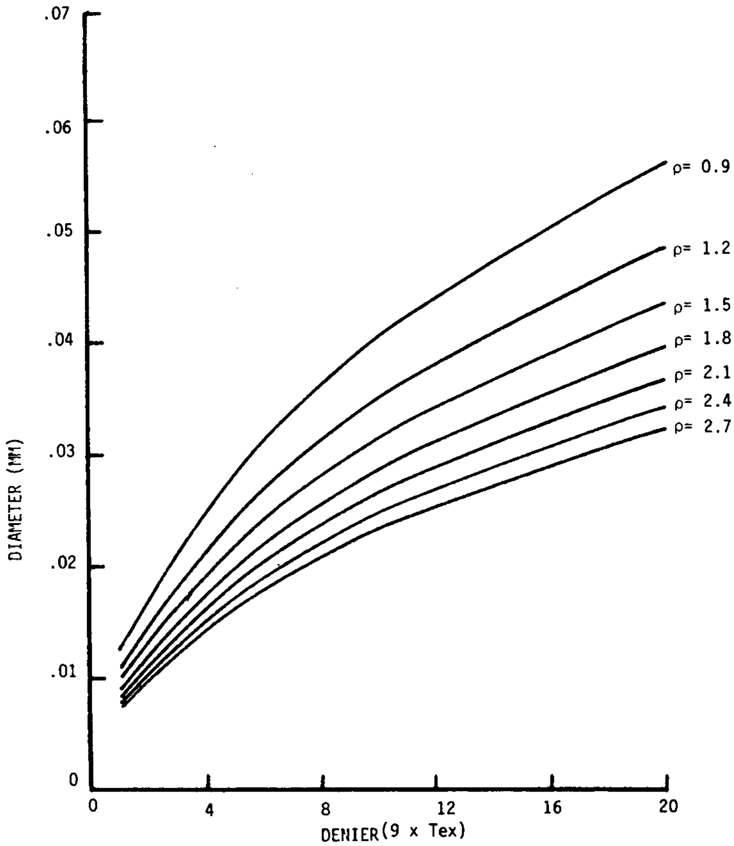

Fig. 1. Diameter of cylindrical fiber vs. denier of fiber for different densities of polymeric material. ρ = density of fiber (g/cc).

made of denser material will acquire smaller diameter for the same denier value. It further shows that decrease in fiber diameter is not linearly proportional to the increase in density of the material.

Tubular Fibers: Three cases of a tubular fiber are analyzed to predict five different properties of the fiber. These three cases of a tubular fiber are shown in Figure 2. All the five properties are predicted in terms of the ratio of diameter of the hollow core of tubular fiber to the diameter of its counterpart circular cylindrical fiber.

The effect of the size of hollow core on five different properties of the tubular fiber are: (1) outside diameter of tubular fiber; (2) wall thickness of tubular fiber; (3) change in weight/unit length of fiber; (4) change in volume/unit length of fiber; (5) change in rigidity of fiber.

Case I. Constant Outside Diameter

The outside diameters of a cylindrical fiber and of the tubular fiber are assumed to be same. If D is the diameter of cylindrical fiber, then the outside diameter, d_0, and wall thickness t of the tubular fiber can be written as

$$d_0/D = 1.0 \tag{6}$$

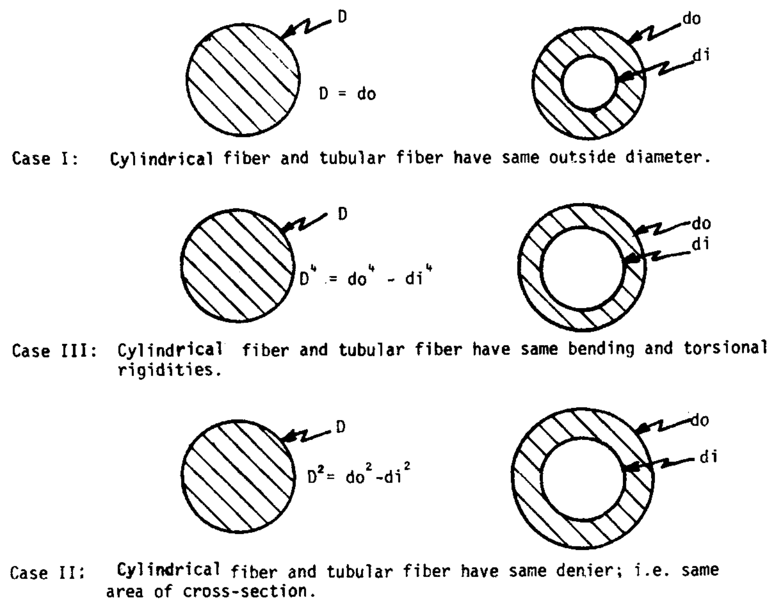

Fig. 2. Three cases of a tubular fiber as compared to cylindrical fiber.

and

$$t/D = \tfrac{1}{2}(1 - d_i/D) \tag{7}$$

where d_i is the inner diameter of tubular fiber. The above equation is written in dimensionless quantities to eliminate the effect of units in the desired relations.

Percent change in weight/unit length of the tubular fiber with respect to the cylindrical fiber can be calculated by the change in the material area of cross section of the fiber, i.e.,

$$\%\Delta W = \frac{(\pi d_o^2/4 - \pi d_i^2/4) - \pi D^2/4}{\pi D^2/4} \times 100 \tag{8}$$

Substituting eq. (6) in eq. (8) and simplifying gives

$$\%\Delta W = -100(d_i/D)^2 \tag{9}$$

The negative sign in eq. (8) indicates the reduction in weight of tubular fiber with increase in the size of hollow core.

As the tubular and cylindrical fibers have the same outside diameter, the volume/unit length of the tubular fiber will remain the same as that of cylindrical fiber, i.e.,

$$\%\Delta V = 0 \tag{10}$$

Bending and torsional rigidities of a rod are defined by EI and GJ, respectively.[7] As the materials of tubular and cylindrical fiber are assumed to be same, the change of rigidities will be proportional to the change in moment of inertia of the cross section of fibers. For circular cross section, the values of I and J are defined as: for cylindrical fiber,

$$I = \pi D^4/64 \tag{11}$$

$$J = \pi D^4/32 \tag{12}$$

for tubular fiber,

$$I = \pi(d_o^4 - d_i^4)/64 \tag{13}$$

$$J = \pi(d_o^4 - d_i^4)/32 \tag{14}$$

It is obvious from eqs. (11)–(14) that the polar moment of inertia of a circular cross section is twice its rectangular moment of inertia, so the percent change in bending rigidity of a tubular fiber with respect to the cylindrical fiber will be same as percent change in torsional rigidity of the fiber. The change in fiber rigidity due to hollow core can be calculated as

$$\%\Delta R = \frac{\pi(d_o^4 - d_i^4)/64 - \pi D^4/64}{\pi D^4/64} \times 100 \tag{15}$$

Substituting the value of d_o from eq. (6) and rearranging the terms gives

$$\%\Delta R = -100(d_i/D)^4 \tag{16}$$

Case II. Constant Denier

The denier (or weight/unit length) of a tubular fiber is assumed to be same as that of cylindrical fiber. As both the fibers are made of same material, the area of cross section of both the fibers should be same, i.e.,

$$\tfrac{1}{4}\pi D^2 = \tfrac{1}{4}\pi(d_o^2 - d_i^2) \tag{17}$$

i.e.,

$$D^2 = d_o^2 - d_i^2 \tag{18}$$

Outside diameter of the tubular fiber with respect to diameter of cylindrical fiber can be obtained by rearranging the terms in eq. (18), i.e.,

$$d_o/D = \sqrt{1 + (d_i/D)^2} \tag{19}$$

Wall thickness of the tubular fiber can be obtained from

$$t = \tfrac{1}{2}(d_o - d_i) \tag{20}$$

Dividing both sides by D and substituting eq. (19) yields

$$t/D = \tfrac{1}{2}[\sqrt{1 + (d_i/D)^2} - d_i/D] \tag{21}$$

As the tubular and cylindrical fibers are of same denier, the weight/unit length of both the fibers will be same, i.e.,

$$\%\Delta W = 0 \tag{22}$$

The percent increase in volume of tubular fiber due to hollow core will be

$$\%\Delta V = \frac{\tfrac{1}{4}\pi d_o^2 - \tfrac{1}{4}\pi D^2}{\tfrac{1}{4}\pi D^2} \times 100 \tag{23}$$

Substituting the value of d_o from eq. (18) and simplifying gives

$$\%\Delta V = 100(d_i/D)^2 \tag{24}$$

Change in rigidity of fiber due to hollow core is defined by eq. (15), which can be written as

TABLE I
Mathematical Expressions to Determine Different Properties of a Tubular Fiber with Change in the Size of Hollow Core, in Three Different Cases of Tubular Fiber

Property	Case I: constant outer diameter	Case II: constant denier	Case III: constant rigidity
d_o/D	1.0	$\sqrt{1+(d_i/D)^2}$	$\sqrt[4]{1+(d_i/D)^4}$
t/D	$\frac{1}{2}(1-d_i/D)$	$\frac{1}{2}[\sqrt{1+(d_i/D)^2}-d_i/D]$	$\frac{1}{2}[\sqrt[4]{1+(d_i/D)^4}-d_i/D]$
$\%\Delta W$	$-100(d_i/D)^2$	0	$100[\sqrt{1+(d_i/D)^4}-(d_i/D)^2-1]$
$\%\Delta V$	0	$100(d_i/D)^2$	$100[\sqrt{1+(d_i/D)^4}-1]$
$\%\Delta R$	$-100(d_i/D)^4$	$200(d_i/D)^2$	0

$$\%\Delta R = \left(\frac{d_o^4 - d_i^4}{D^4} - 1\right) \times 100 \tag{25}$$

or

$$\%\Delta R = \left[\frac{(d_o^2 + d_i^2)(d_o^2 - d_i^2)}{D^4} - 1\right] \times 100 \tag{26}$$

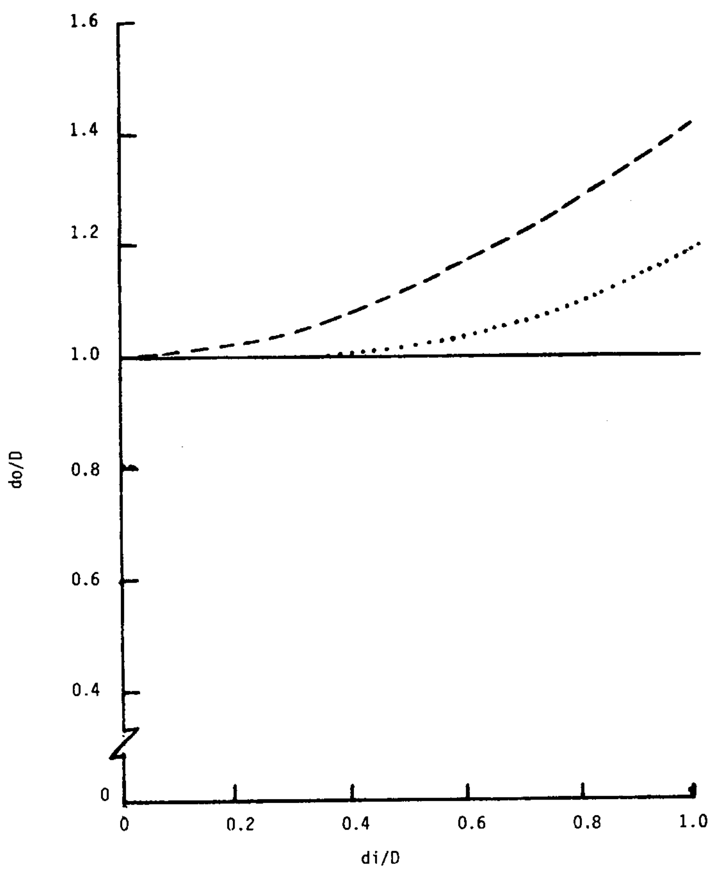

Fig. 3. Change in outside diameter of hollow fiber with increase in size of hollow core: (—) constant outside diameter; (- - -) constant denier; (. . . .) constant rigidity. D = diameter of cylindrical fiber, d_i = inside diameter of hollow core, d_o = outside diameter of tubular fiber.

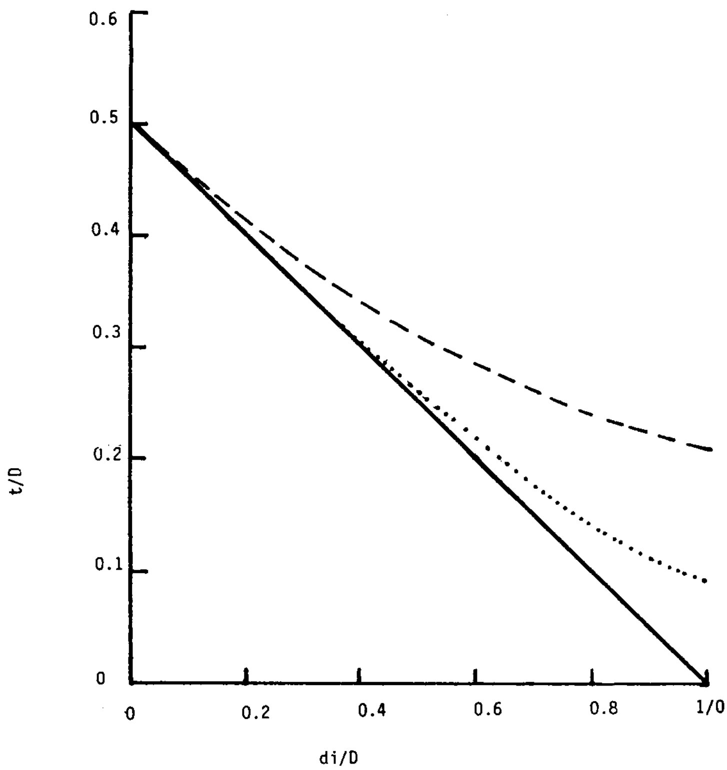

Fig. 4. Change in wall thickness of tubular fiber with increase in size of hollow core: (—) constant outside diameter, (- - -) constant denier; (. . . .) constant rigidity. D = diameter of cylindrical fiber, d_i = inside diameter of hollow core, and t = wall thickness of tubular fiber.

Using eq. (18), the above equation can be reduced to

$$\%\Delta R = 200(d_i/D)^2 \tag{27}$$

Case III. Constant Rigidity

The bending and torsional rigidity of a tubular fiber are assumed to be same as that of cylindrical fiber. It implies that both the fibers have same moment of inertia, i.e.,

$$\%\Delta R = 0 \tag{28}$$

and

$$\pi D^4/64 = \pi(d_o^4 - d_i^4)/64 \tag{29}$$

or

$$D^4 = d_o^4 - d_i^4 \tag{30}$$

i.e.,

$$d_o/D = \sqrt[4]{1 + (d_i/D)^4} \tag{31}$$

The wall thickness of tubular fiber can be found by substituting the value of d_o from eq. (31) into eq. (20). It gives

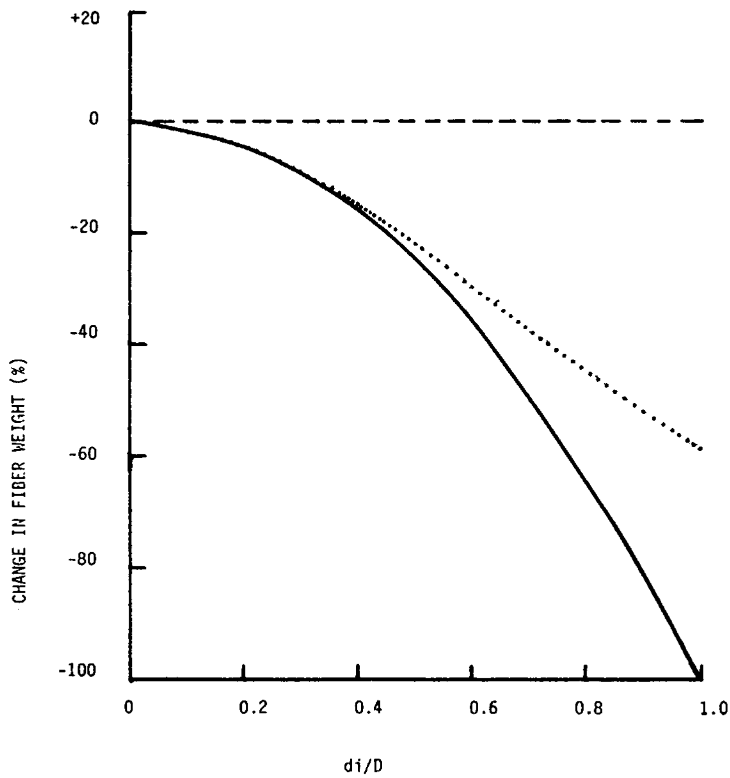

Fig. 5. Percent reduction in fiber weight with increase in size of hollow core: (—) constant outside diameter; (- - -) constant denier; (. . . .) constant rigidity. D = diameter of cylindrical fiber, and d_i = inside diameter of hollow core.

$$t/D = \tfrac{1}{2}[\sqrt[4]{1 + (d_i/D)^4} - d_i/D] \tag{32}$$

Change in weight/unit length and volume/unit length of tubular fiber can be found by using eqs. (8) and (23), respectively, and eliminating the term d_o in them by using eq. (31). This gives

$$\%\Delta W = 100[\sqrt{1 + (d_i/D)^4} - (d_i/D)^2 - 1] \tag{33}$$

$$\%\Delta V = 100[\sqrt{1 + (d_i/D)^4} - 1] \tag{34}$$

DISCUSSION

The equations developed to predict the five properties, for three different cases of a tubular fiber, are summarized in Table I and plotted in Figures 3–7.

As shown in Table I, all the equations derived are in dimensionless quantities, and each property is defined in terms of the ratio of the inner diameter of tubular fiber to the diameter of its corresponding cylindrical fiber. It further shows that for the case of constant outside diameter of tubular fiber, increase in the size of hollow core reduces the fiber weight by only the second order of the diameter of the inner hole while it reduces the fiber rigidity by the fourth order of the diameter of the inner hole. In the case of constant denier tubular fiber, the fiber

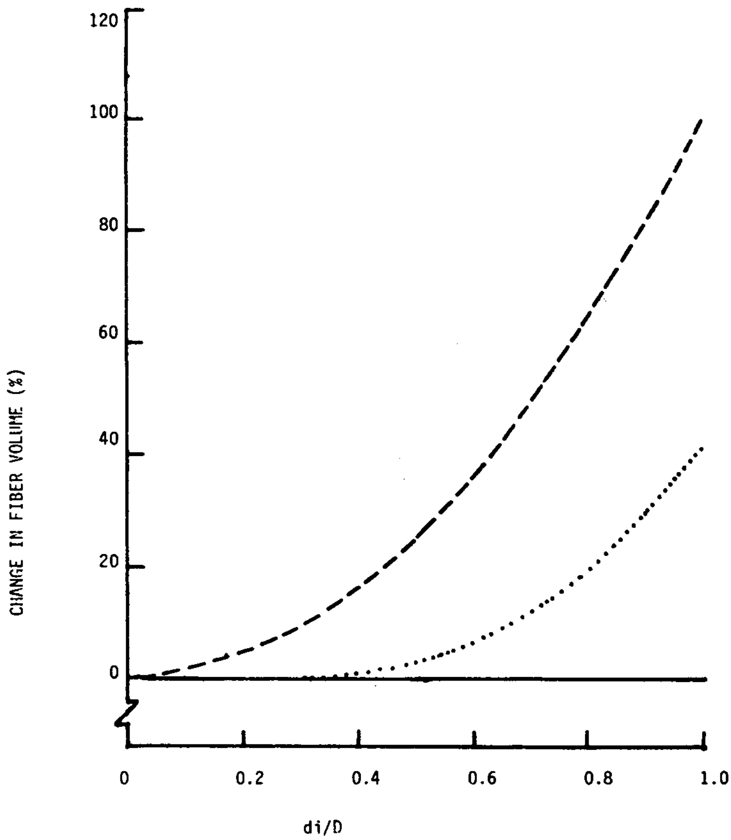

Fig. 6. Percent increase in fiber volume with increase in size of hollow core: (—) constant outside diameter; (- - -) constant denier; (. . . .) constant rigidity. D = diameter of cylindrical fiber and d_i = inside diameter of hollow core.

volume increases by the second order of the size of the inner hole and percent increase in fiber rigidity is twice the percent increase in fiber volume.

Figure 3 compares the three cases of tubular fiber for its outer diameter. It shows that the outside diameter of tubular fiber of constant denier increases more rapidly with increase in the size of the hollow core as compared to the outside diameter of tubular fiber of same rigidity.

Figure 4 shows that the increase in the size of the hollow core will linearly decrease the wall thickness of tubular fiber of constant outside diameter. The cases of constant rigidity and constant denier show concave upward relations for the wall thickness of tubular fiber. It further shows that, up to 40% of the size of the inner hole as compared to the outside diameter of tubular fiber, the wall thickness of constant rigidity and constant outside diameter tubular fibers remains almost same and further increase in size of the inner hole will cause thicker wall for constant rigidity tubular fiber.

Change in weight/unit length of tubular fiber with change in the size of hollow core is shown in Figure 5. It shows more reduction in fiber weight of constant outside diameter tubular fiber as compared to the case of constant rigidity tubular fiber for larger sizes of hollow core, but, up to 40% of the size of inner hole, both the fibers have almost the same reduction in fiber weight.

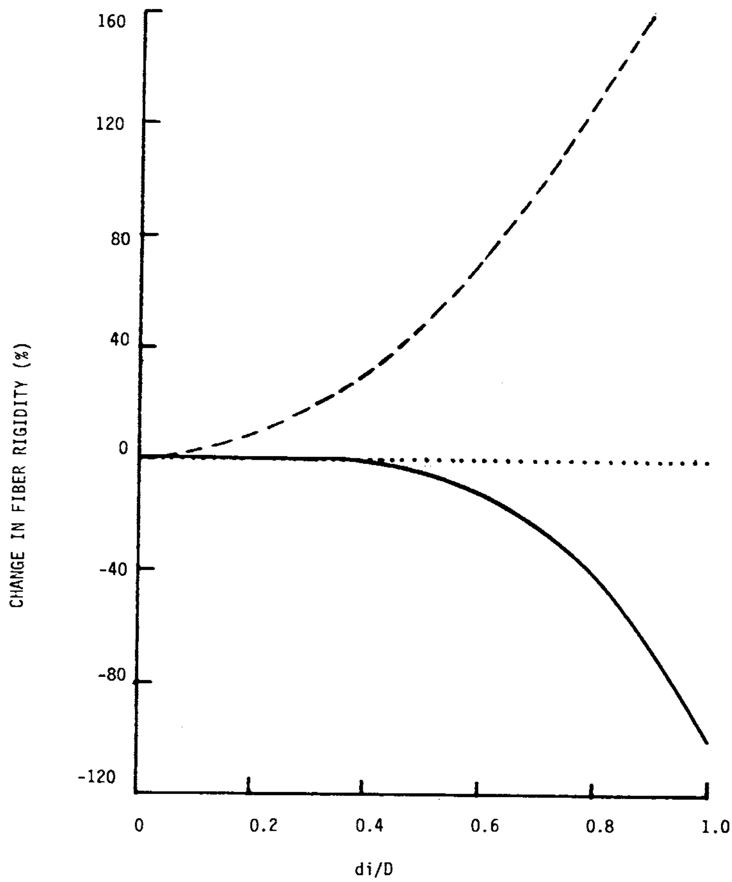

Fig. 7. Percent change in fiber rigidity with increase in size of hollow core: (—) constant outside diameter; (---) constant denier; (. . . .) constant rigidity. D = diameter of cylindrical fiber and d_i = inside diameter of hollow core.

Figure 6 shows the plot for increase in fiber volume with increase in the size of hollow core. Constant denier tubular fiber shows more rapid increase in fiber volume as compared to constant rigidity tubular fiber for the same size of hollow core.

Figure 7 shows that constant denier tubular fiber will have increase in fiber rigidity while constant outer diameter tubular fiber will lose its rigidity with increase in the size of hollow core. Rigidity is the measure of flexibility, and so is hand of the fiber. Change in fiber rigidity will affect the compressibility and so the comfort properties of the product. If the aim of producing tubular fiber is to reduce fiber weight and increase fiber volume without affecting the fiber stiffness property, then the proper dimensions of the tubular fiber must be chosen from the constant rigidity case. If the purpose is only to reduce fiber weight or only to increase fiber volume, then the case of constant outside diameter tubular fiber or the case of constant denier tubular fiber should be considered, respectively. Of course, it would affect the fiber rigidity property.

Comparing Figures 3–7, it can be observed that, up to 40% of the size of the hollow core, compared to the outer diameter of the fiber, there is no significant difference in the property of constant rigidity and constant outer diameter tubular fibers. Figure 5 shows that by having a hollow core of 0.4 the size of outer dimension of a fiber, can save up to 16% in material weight. It further shows that a fiber having a hollow core of 0.5 its outer diameter has only 6% reduction in fiber rigidity, for the case of constant outside diameter tubular fiber, but it can save as much as 25% in material weight. This 6% loss in fiber rigidity can be totally recovered by increasing the outer dimension of fiber by 1.5% and sacrificing saving in material weight by only 3%.

The theory developed here can be verified by taking the microphotograph of a hollow fiber cross section for its dimensions and measuring the bending and torsional rigidity of the fiber by utilizing the principle of transverse and torsional vibrations, respectively. A vibroscope, which is generally used to measure the linear density of a fine fiber, works on the principle of transverse vibration. This instrument can be modified to measure the bending rigidity of a hollow fiber. Torsional rigidity of a hollow fiber can be obtained by making a torsional pendulum of the fiber sample and checking its natural frequency of torsional vibration.

CONCLUSIONS

The theory developed here could be used for determining the dimension of a tubular fiber to comply with specific end use of the product, in terms of its weight and stiffness property. Even though the edge effect of a hollow fiber, when it is extruded, is not considered in the analysis, it can be assumed that the difference in the edge effect of the two hollow fibers of comparable dimensions, on the rigidity and other properties of the fibers, is negligible. Furthermore, the model analyzed here is for a single fiber, but it is believed that it can be used to compare the relative behavior of different tubular fibers in bundle form, where interfiber friction plays an important role.

APPENDIX: NOMENCLATURE

D	diameter of a cylindrical fiber (mm)
d_o	outside diameter of tubular fiber
d_i	inside diameter of tubular fiber
n_d	denier of fiber
n_t	tex of fiber
I	rectangular moment of inertia of fiber cross section
J	polar moment of inertia of fiber cross section
E	elastic modulus of fiber material
G	shear modulus of fiber material
t	wall thickness of tubular fiber
w	weight of 1 cm long fiber
$\%\Delta W$	percent change in fiber weight
$\%\Delta V$	percent change in fiber volume
$\%\Delta R$	percent change in fiber rigidity
ρ	density of fiber material (g/cc)

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